

Sec 2: DSP

~~Example~~

a) $y(n) = n x(n)$

$$x_1(n) \rightarrow n x_1(n)$$

$$x_2(n) \rightarrow n x_2(n)$$

$$x_1(n) + x_2(n) \rightarrow n(x_1(n) + x_2(n)) = y_1 + y_2$$

\rightarrow Linear

\rightarrow output depends only on present i/p \rightarrow static

\rightarrow o/p doesn't depend on future i/p \rightarrow Causal

* $y(n, K) = n x(n-K)$

$$y(n-K) = (n-K) x(n-K)$$

$$y(n-K) \neq y(n, K) \rightarrow \text{shift variant}$$

\rightarrow unstable system.

\rightarrow input multiplied with n

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$$b) y(n) = a x(n)$$

→ Linear

→ static

→ Causal

→ shift-invariant.

→ stable

$$c) y(n) = x(n^2)$$

$$\rightarrow x_1(n) \rightarrow x_1(n^2)$$

$$x_2(n) \rightarrow x_2(n^2)$$

$$x_1(n) + x_2(n) \rightarrow \underbrace{x_1(n^2) + x_2(n^2)}_{y_3} = y_1 + y_2$$

* Dynamic.

$$y(5) = y(25)$$

↑
Future

* non-Causal.

$$* y(n, k) = x(n^2 - k)$$

$$y(n - k) = x((n - k)^2)$$

$\neq \Rightarrow$ shift variant

* stable

$$y(n) = x(n) + 3u(n+1)$$

$$y_1(n): x_1(n) \rightarrow x_1(n) + 3u(n+1)$$

$$y_2(n): x_2(n) \rightarrow x_2(n) + 3u(n+1)$$

$$x_1(n) + x_2(n) \rightarrow x_1(n) + x_2(n) + 3u(n+1)$$

$$\rightarrow y_3(n) \neq y_1(n) + y_2(n) \quad \text{non-linear}$$

\rightarrow Dynamic (non-causal)

$$\Rightarrow y(n, K) = x(n-K) + 3u(n+1)$$

$$y(n-K) = x(n-K) + 3u(n-K+1)$$

$$y(n, K) \neq y(n-K) \rightarrow \text{shift variant}$$

\rightarrow stable system

Convolution

$$y(n) = \sum_{K=-\infty}^{\infty} x(K) \cdot h(n-K)$$

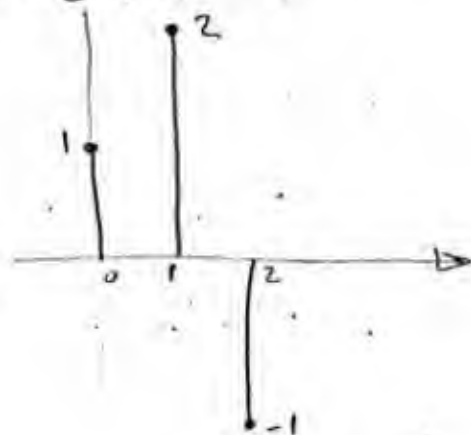
$$* \quad x(n) = \{1, 2, -1\}, \quad h(n) = \{1, 2, -1\}$$

$$\underline{n=0}$$

$$y(0) = \sum_0^2 x(k) h(k)$$

$$= x(0)h(0) + x(1)h(-1) + x(2)h(-2)$$

$$y(0) = 1$$



$$\underline{n=1} \quad y(1) = \sum_0^2 x(k) h(1-k)$$

$$= x(0)h(1) + x(1)h(0) = 2 + 2 = 4$$

$$\underline{n=2}$$

$$y(2) = \sum_0^2 x(k) h(2-k)$$

$$= x(0)h(2) + x(1)h(1) + x(2)h(0)$$

$$= -1 + 4 - 1 = 2$$

$$\underline{n=3} \quad y(3) = \sum_1^2 x(k) h(3-k)$$

$$= x(1)h(2) + x(2)h(1) = -2 - 2 = -4$$

$$\begin{aligned} \underline{n=4} \quad y(4) &= \sum_{k=0}^2 x(k) h(4-k) \\ &= x(2) h(2) = 1 \end{aligned}$$

$$y(5) = \sum x(k) h(n-k) \text{ so}$$

$$y = \{1, 4, 2, -4, 1\}$$

$$x_{\min} + h_{\min} \leq n \leq x_{\max} + h_{\max}$$

$$* \quad y(n) = x(n) + 0.5x(n-1) + 0.25x(n-2)$$

From step input Find $y(n)$ using Convolution

$$\begin{aligned} \underline{n=0} \quad y(0) &= x(0) + 0.5x(-1) + 0.25x(-2) \\ &= x(0) = 1 \end{aligned}$$

$$\begin{aligned} \underline{n=1} \quad y(1) &= x(1) + 0.5x(0) + 0.25x(-1) \\ &= 1 + 0.5 = 1.5 \end{aligned}$$

$$\begin{aligned} \underline{n=2} \quad y(2) &= x(2) + 0.5x(1) + 0.25x(0) \\ &= 1.75 \end{aligned}$$

$$y(n) = \sum h(n-k)$$

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